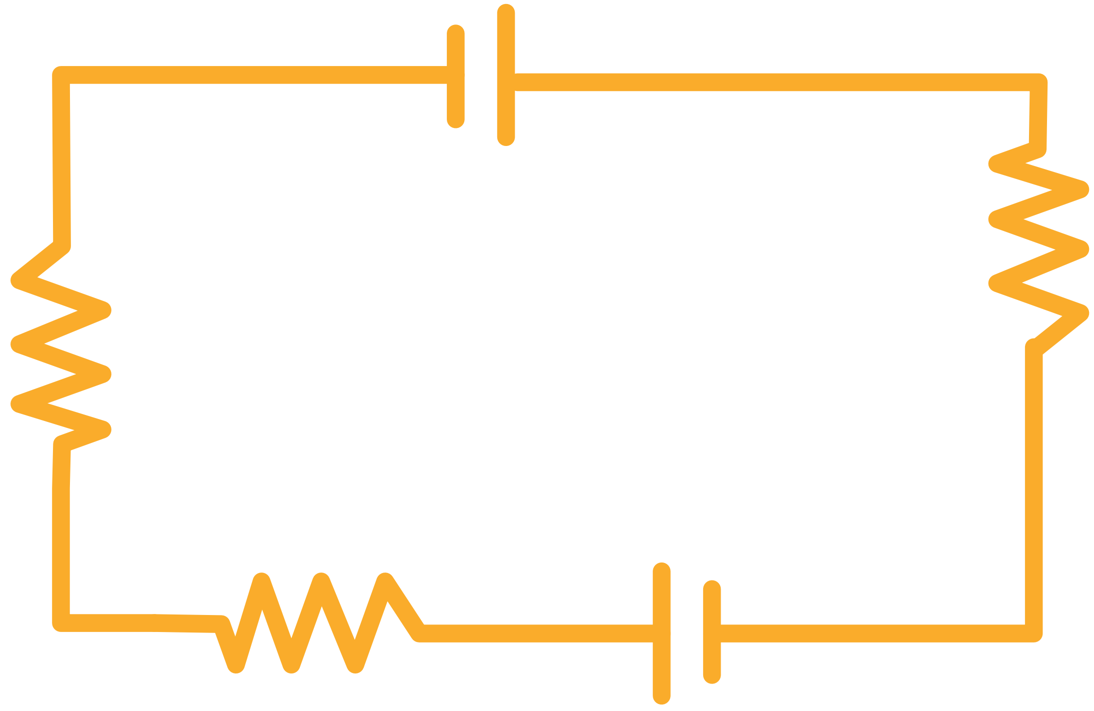
Energy Conservation in Circuits



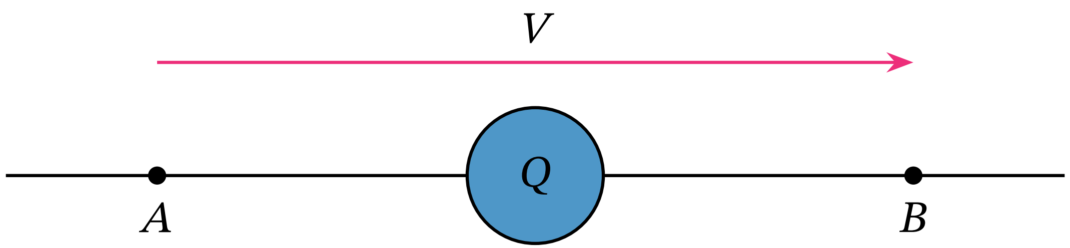
Lesson Objectives

You will be able to

- use the fact that the current flowing into a circuit junction equals the current flowing out of the same circuit junction to determine currents in circuit branches,
- use the fact that the sum of the emfs of voltage sources across a circuit loop equals the sum of the potential drop across the components in the loop to determine emfs and potential drops.

Recap of Electric Energy

Consider two points in a circuit, point A and point B, with potential difference V across them caused by an electric field such that point A is at a higher potential than point B. Now, consider a positive charge, Q, placed at point A. The potential difference will cause the charge to move from point A to point B, as illustrated in the following diagram.



The charge moves because the potential difference causes a force to act on the charge, which does work on it as it moves. The work done on the charge, E, is equal to the charge multiplied by the potential difference between the two points:

$$E = QV.$$

Recap of Electric Energy (Continued)

As the charge moves through from higher to lower potential, energy is transferred from electrical potential energy to other categories of energy.

In an electric circuit, we can consider the amount of charge, Q, that has moved past a point over some time, t. Recall that the current in an electric circuit, I, is equal to the amount of charge that has passed the point in the circuit that is being measured divided by time:

$$I = \frac{Q}{t}.$$

This can be rearranged to give an expression for the charge that has passed this point in the circuit:

$$Q = It.$$

We can substitute this equation, $Q = It$, back into our equation for electrical energy, $E = QV$, such that

$$E = ItV.$$

Recap of Electric Energy (Continued)

Then, we can divide both sides of the equation by t:

$$\frac{E}{t} = IV.$$

This is the amount of energy per unit of time that the electric circuit consumes—also known as power:

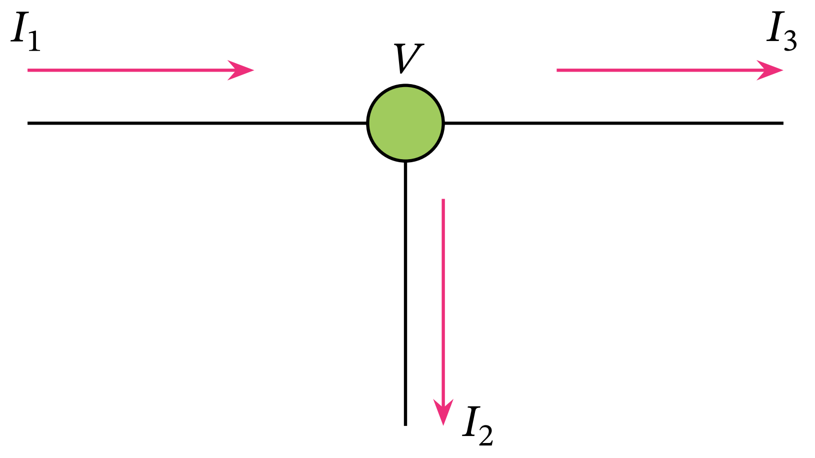
$$P = IV.$$

Energy Conservation in Electric Circuits

In an electric circuit, energy is conserved; the circuit cannot spontaneously create or destroy energy. Electrical energy can, however, be transformed to other categories of energy between points where the potential of the circuit changes (e.g., across a resistor).

Let us consider a point in a circuit where the circuit splits in two. We call the point at which a circuit splits or joins together a "node," or a "junction."

The diagram below shows this point in a circuit with current I_1 into the node and I_2 and I_3 out of the node.



Energy Conservation in Electric Circuits (Continued)

The amount of energy into the node per unit of time, the power in, is equal to the sum of currents into the node multiplied by the voltage of the node, V:

$$P_{\text{in}} = I_1 V.$$

The amount of energy out of the node per unit of time, the power out, is equal to the sum of currents out of the node multiplied by the voltage of the node:

$$P_{\text{out}} = (I_2 + I_3) V.$$

Energy at the node is conserved; the power into the node is equal to the power out of the node:

$$P_{\text{in}} = P_{\text{out}}.$$

This can be expressed in terms of current and potential as

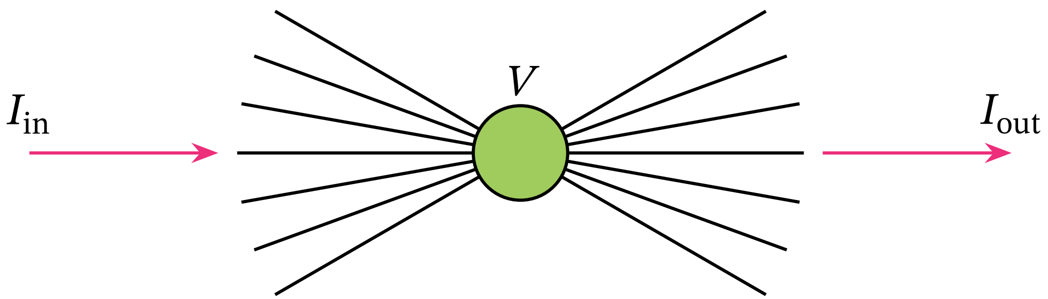
$$I_1 V = (I_2 + I_3) V.$$

We can divide both sides of this equation by V to give an expression for the currents in and out of the node:

$$I_1 = I_2 + I_3.$$

Energy Conservation in Electric Circuits (Continued)

Now, imagine a node with many currents in and many out.



Again, we can calculate the power into the node:

$$P_{in} = \left(I_{1,in} + I_{2,in} + \cdots\right) V.$$

Similarly, we can calculate the power out of the node:

$$P_{out} = \left(I_{1,out} + I_{2,out} + \cdots\right) V.$$

Energy Conservation in Electric Circuits (Continued)

Energy is conserved, so

$$P_{\text{in}} = P_{\text{out}}.$$

Substituting the expressions for P_{in} and P_{out},

$$\left(I_{1,\text{in}} + I_{2,\text{in}} + \cdots\right) V = \left(I_{1,\text{out}} + I_{2,\text{out}} + \cdots\right) V,$$

and then dividing both sides by V give an expression relating the currents in and out of a node:

$$I_{1,\text{in}} + I_{2,\text{in}} + \cdots = I_{1,\text{out}} + I_{2,\text{out}} + \cdots .$$

This is the first of Kirchhoff's two laws. Kirchhoff's first law states that the sum of the currents into a node equals the sum of the currents out of a node.

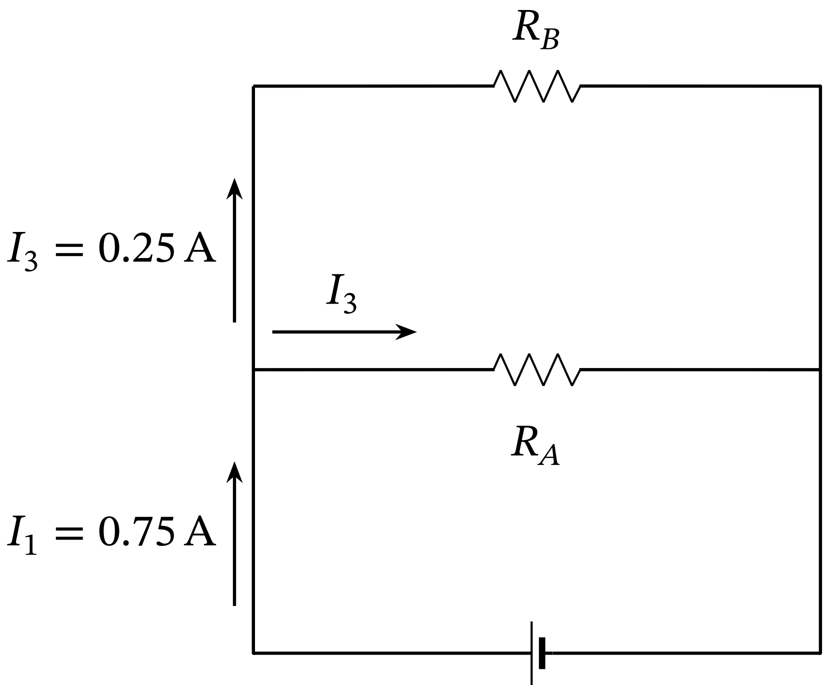
Definition: Kirchhoff's First Law

The sum of the currents into a node in a circuit, $I_{1,\mathrm{in}} + I_{2,\mathrm{in}} + \cdots$, equals the sum of the currents out of the node, $I_{1,\mathrm{out}} + I_{2,\mathrm{out}} + \cdots$:

$$I_{1,\mathrm{in}} + I_{2,\mathrm{in}} + \cdots = I_{1,\mathrm{out}} + I_{2,\mathrm{out}} + \cdots .$$

Illustrative Example of Kirchhoff's First Law

This can be used in circuits to calculate currents in each branch of a circuit. For example, in the following diagram, the currents in two branches are known and can be used to calculate the current in the third branch.



Illustrative Example of Kirchhoff's First Law (Continued)

Without knowing the value of either resistor or even the potential difference across the battery, we can use Kirchhoff's first law to relate the three currents:

$$I_1 = I_2 + I_3.$$

Rearranging this for I_3 and substituting the values $I_1 = 0.75\ \text{A}$ and $I_2 = 0.25\ \text{A}$, we get

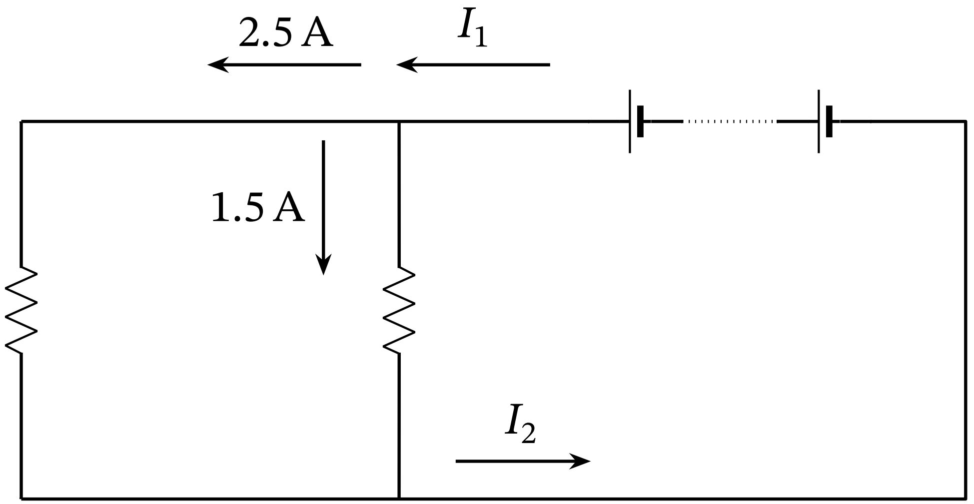
$$I_3 = I_1 - I_2$$

$$I_3 = 0.75\ \text{A} - 0.25\ \text{A}$$

$$I_3 = 0.5\ \text{A}.$$

Example 1: Using Kirchhoff's First Law to Calculate Current in a Circuit

The currents in two wires of the circuit shown are known. The currents I_1 and I_2 are unknown.



1. Find I_1.
2. Find I_2.

Example 1 (Continued)

Answer

Part 1

Kirchhoff's first law states that the sum of currents into a node in a circuit is equal to the sum of currents out of the node. To find I_1, we can consider the node at the top of the circuit. The total current into the node, I_{in}, is equal to

$$I_{in} = I_1,$$

and the total current out of the node, I_{out}, is equal to

$$I_{out} = 2.5\ \text{A} + 1.5\ \text{A}.$$

Equating the two, we get

$$I_1 = 2.5\ \text{A} + 1.5\ \text{A}$$

$$I_1 = 4.0\ \text{A}.$$

Example 1 (Continued)

Part 2

To find I_2, we can consider the node at the bottom of the circuit. The total current into this node, I_{in}, is

$$I_{in} = 2.5\,\text{A} + 1.5\,\text{A},$$

and the current out of this node, I_{out}, is

$$I_{out} = I_2.$$

Equating the two, we get

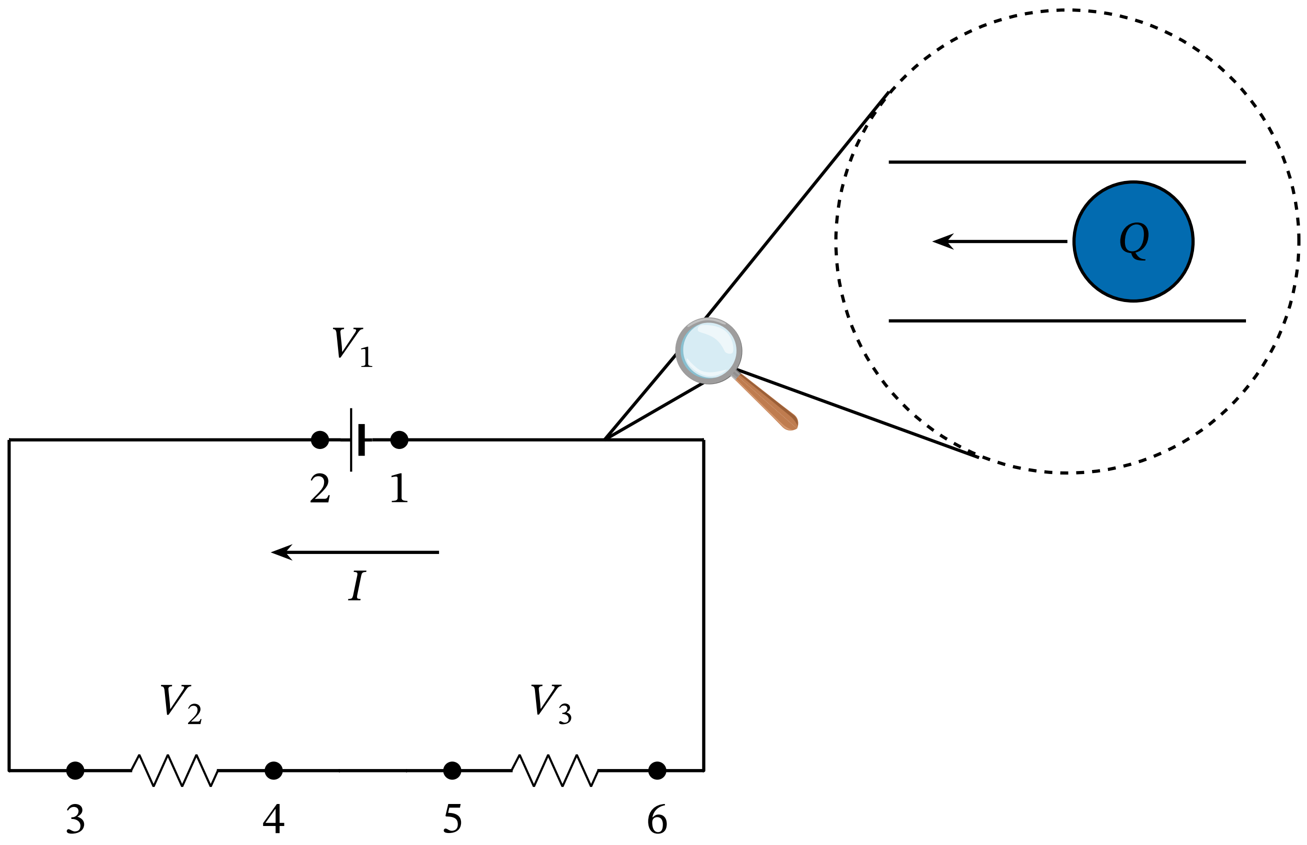
$$2.5\,\text{A} + 1.5\,\text{A} = I_2$$

$$I_2 = 4.0\,\text{A}.$$

Note that I_2 could have been found by looking at the right branch of the circuit and noticing that $I_1 = I_2$ because the currents are at two points in series.

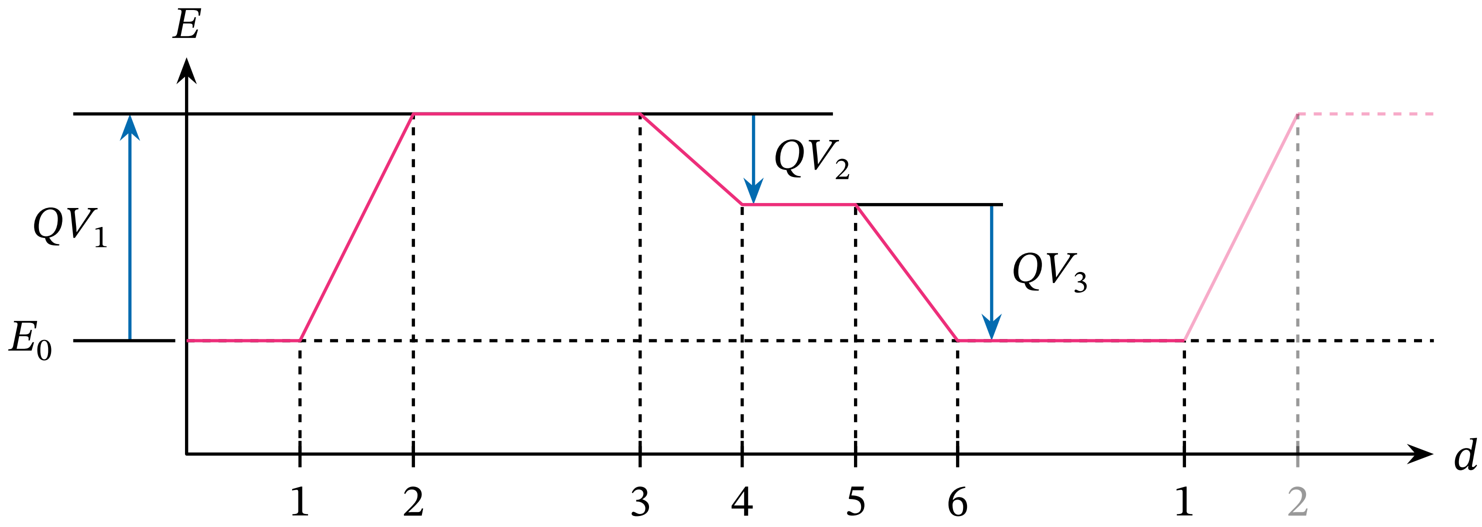
Electric Potential Difference

Take a look at the following circuit, comprising a battery and two resistors. The potential difference across the battery is equal to V_1 and that across the first and second resistors is V_2 and V_3 respectively. We can imagine looking at a zoomed-in view of a positive charge, Q, moving around the circuit. In this case, we will follow a positive charge moving in the direction of the current.



Electric Potential Difference (Continued)

We can calculate the work done on the positive charge as it moves between points 1 to 6 of the circuit. The work done is equal to the magnitude of the charge multiplied by the potential difference across the points it travels between. This is equal to the change in the electrical potential energy of the charge and can be visualized on the following graph of electrical potential energy, E, against distance traveled around the circuit, d.



Electric Potential Difference (Continued)

Note that the work done on the charge only tells us the *change* in electrical potential energy; the charge has some base level of electrical potential energy of E_0.

As seen, the electrical potential energy of the charge by the time it gets back to point 1 of the circuit must be the same as the electrical potential energy of the charge the previous time it was at point 1. This is because energy in the circuit is conserved; within each loop of the circuit, the particle cannot gain or lose electrical potential energy.

This means that the total work done on the charge over a single loop of the circuit must equal zero:

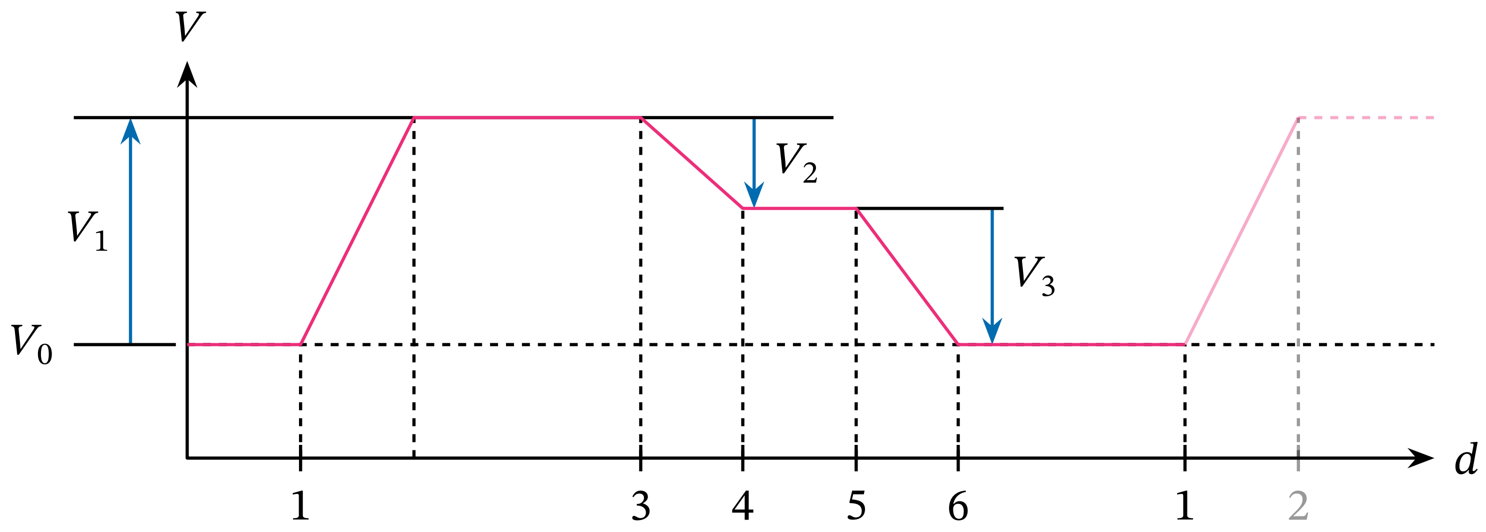
$$QV_1 + QV_2 + QV_3 = 0.$$

The magnitude of the charge is constant, so we can divide this equation by Q:

$$V_1 + V_2 + V_3 = 0.$$

Electric Potential Difference (Continued)

Similarly to electrical potential energy, we can also plot a graph of electrical potential of the charge, V, against distance traveled around the circuit, d.



Note that there is some base-level electrical potential of the charge, V_0; we can only measure potential *difference* between two points in the circuit.

Electric Potential Difference (Continued)

This means that because energy is conserved, around a loop in a circuit, the sum of the potential differences across each component in the loop must equal zero:

$$V_1 + V_2 + \cdots + V_N = 0.$$

This is Kirchhoff's second law; the sum of the potential differences across all components in a loop in a circuit is equal to zero.

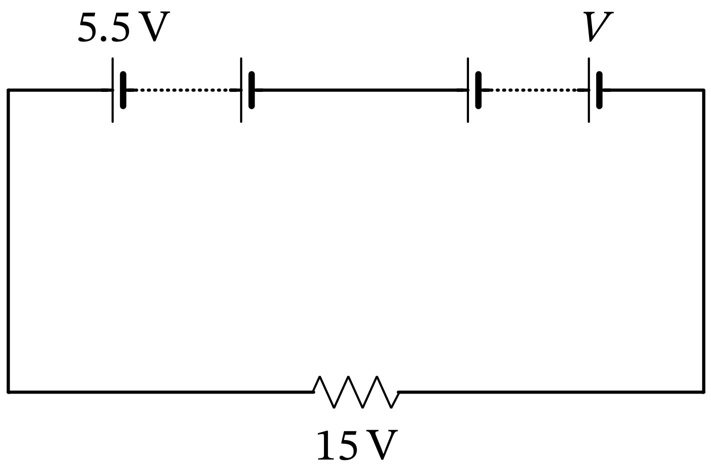
Definition: Kirchhoff's Second Law

The sum of the potential difference across each component in a loop in a circuit is equal to zero:

$$V_1 + V_2 + \cdots + V_N = 0.$$

Example 2: Using Kirchhoff's Second Law to Calculate Voltage in a Circuit

The potential drop across the resistor in the circuit shown is 15 V. The terminal voltage of one of the batteries powering the circuit is 5.5 V. Find the terminal voltage V of the other battery powering the circuit.



Example 2 (Continued)

Answer

Kirchhoff's second law states that the sum of the potential difference across each component in a loop in a circuit is equal to zero.

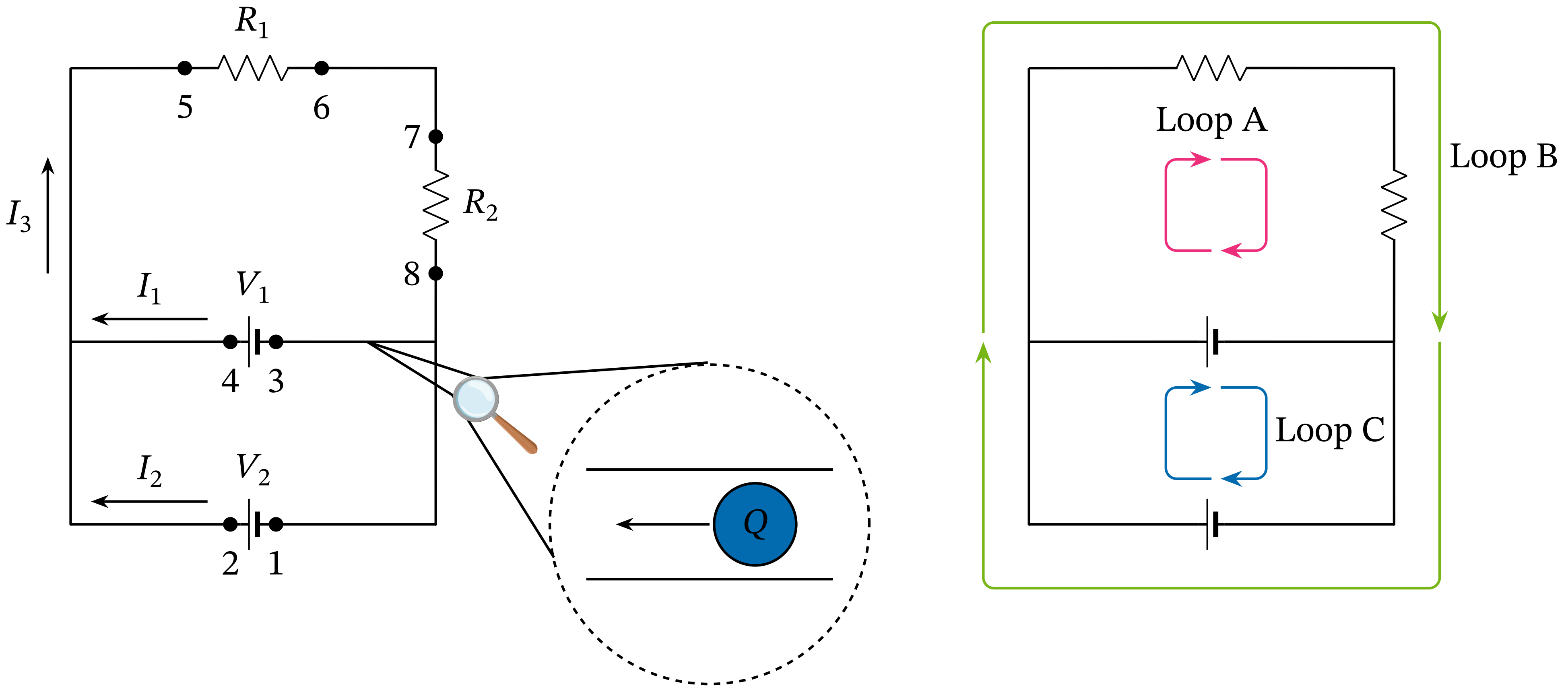
In this circuit, there is a known 5.5 V increase in potential across the first battery, a V gain across the second battery, and a 15 V decrease in potential across the resistor.

Writing Kirchhoff's second law for this loop, we have

$$5.5\,\mathrm{V} + V - 15\,\mathrm{V} = 0\,\mathrm{V}$$
$$V = 9.5\,\mathrm{V}.$$

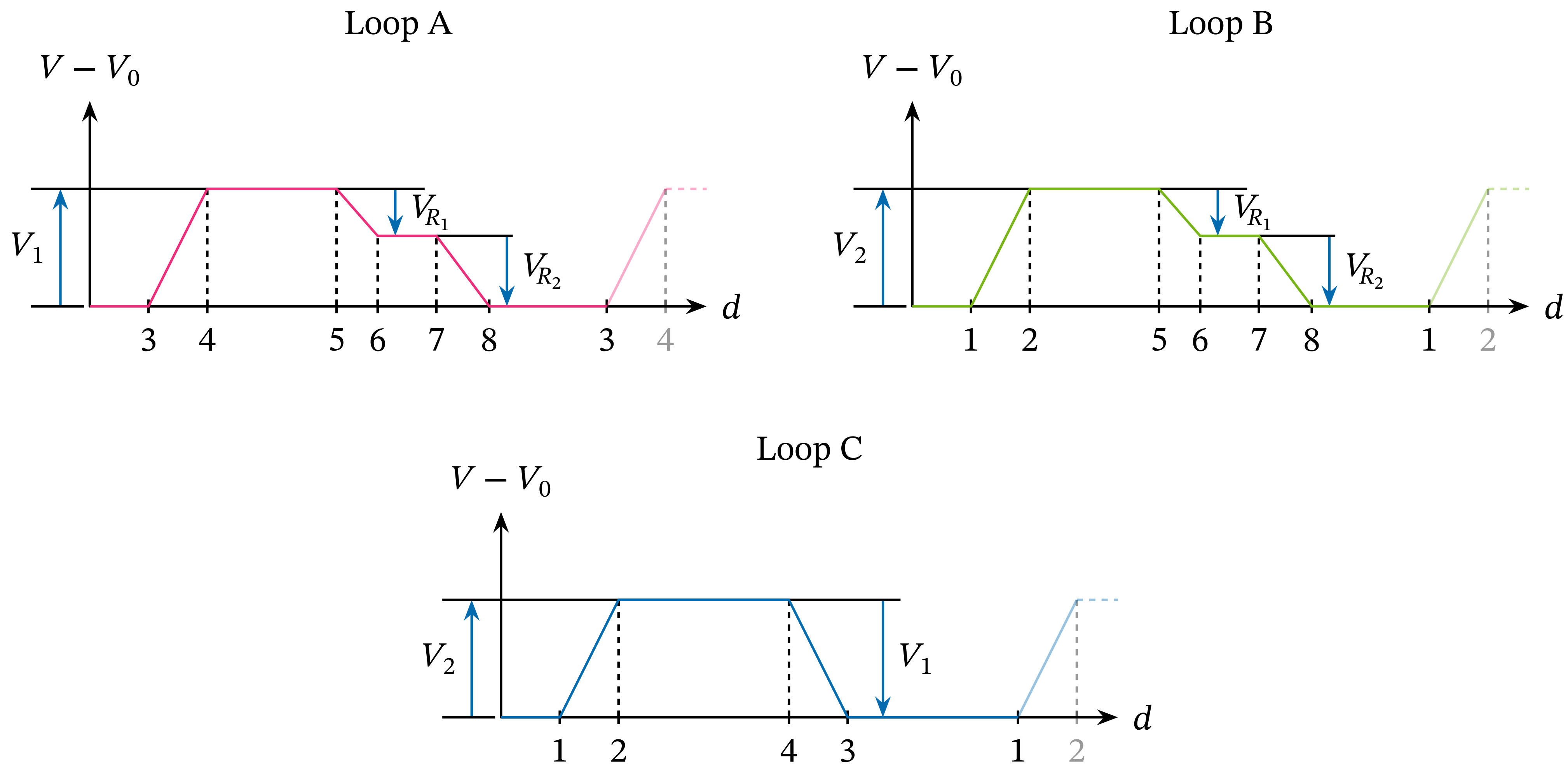
Illustrative Example of Multiple-Branch Circuits

We can apply Kirchhoff's second law to circuits that split into multiple branches. For example, the following circuit has three loops that a charge could take, as shown in the following diagram.



Illustrative Example of Multiple-Branch Circuits (Continued)

Here, we will say that $V_1 = V_2$. We can follow a charge moving in either direction around each loop, and graphs of electrical potential relative to base-level electrical potential, $V - V_0$, against distance traveled around the loop, d, can be plotted. These graphs are shown in the following diagram.

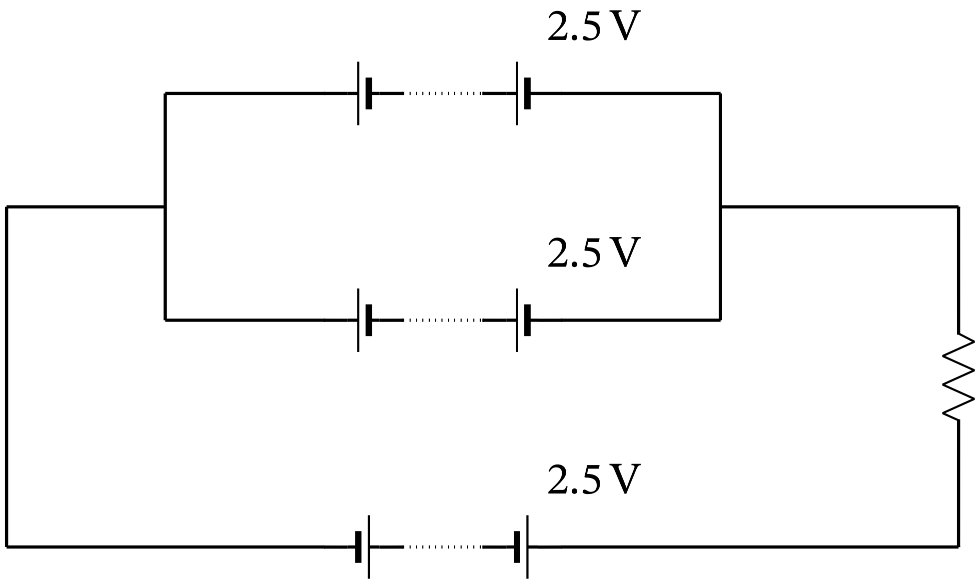


Illustrative Example of Multiple-Branch Circuits (Continued)

As before, across a whole loop, the work done on the charge, and therefore the total potential difference across each component in the loop, must equal zero. This means that Kirchhoff's second law can be applied to any loop in a circuit.

Example 3: Using Kirchhoff's Second Law to Calculate Voltage in a Circuit with Multiple Loops

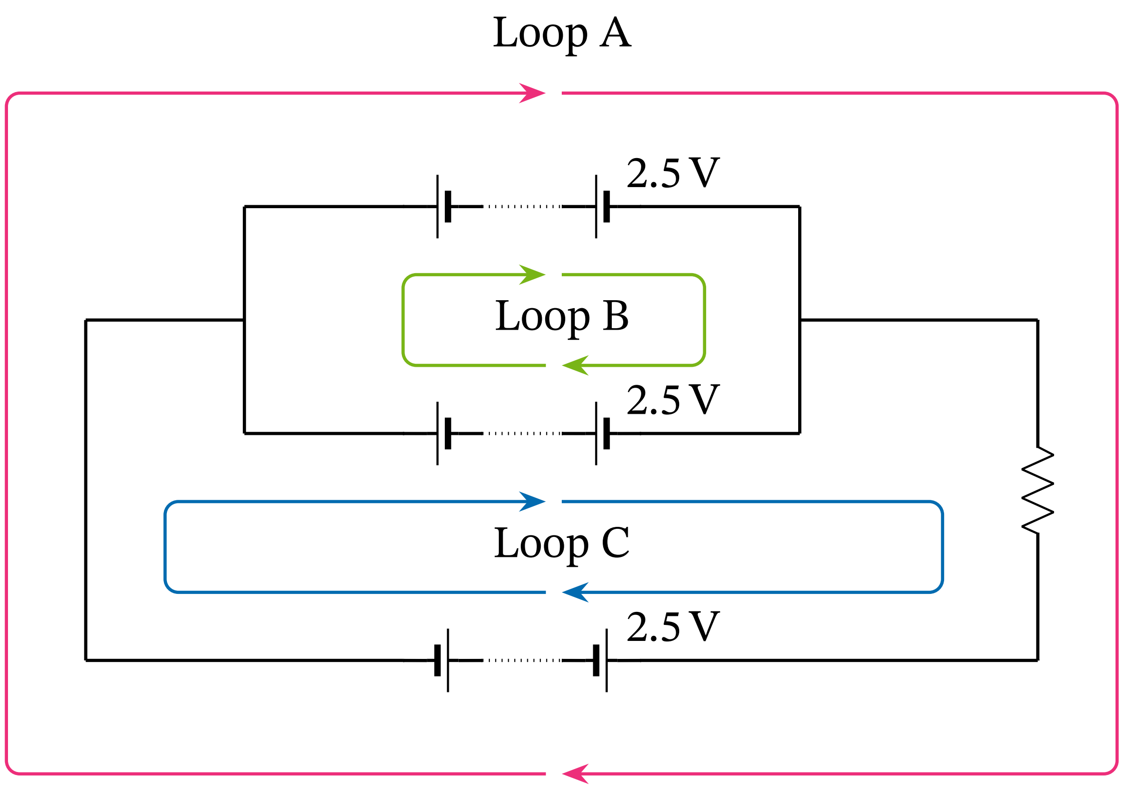
Find the decrease in potential across the resistor in the circuit shown. The batteries powering the circuit each have a terminal voltage of 2.5 V.



Example 3 (Continued)

Answer

This circuit has three loops that can be followed in either direction, which we can label on the circuit diagram.



Example 3 (Continued)

To find the decrease in potential across the resistor, V_R, we can look at either loop A or loop C.

Recall that, according to Kirchhoff's second law, the sum of the potential differences across each component in a loop is equal to zero:

$$V_1 + V_2 + \cdots + V_N = 0.$$

For loop A, this is

$$2.5\,\text{V} + V_R + 2.5\,\text{V} = 0\,\text{V}.$$

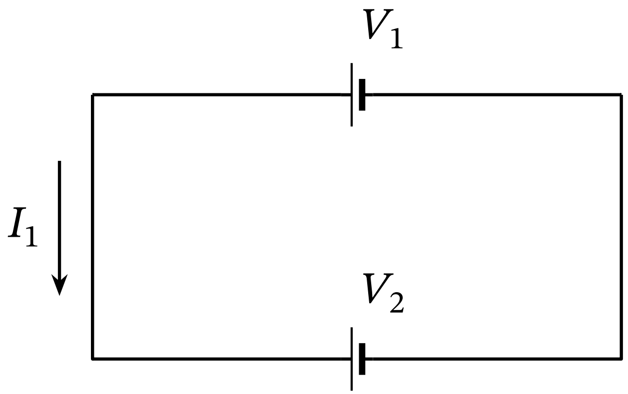
Rearranging gives

$$V_R = -5.0\,\text{V}.$$

So, the decrease in potential across the resistor is equal to 5 V.

Kirchhoff's Second Law Applied to Batteries in Parallel

Let us consider the circuit shown in the following diagram.

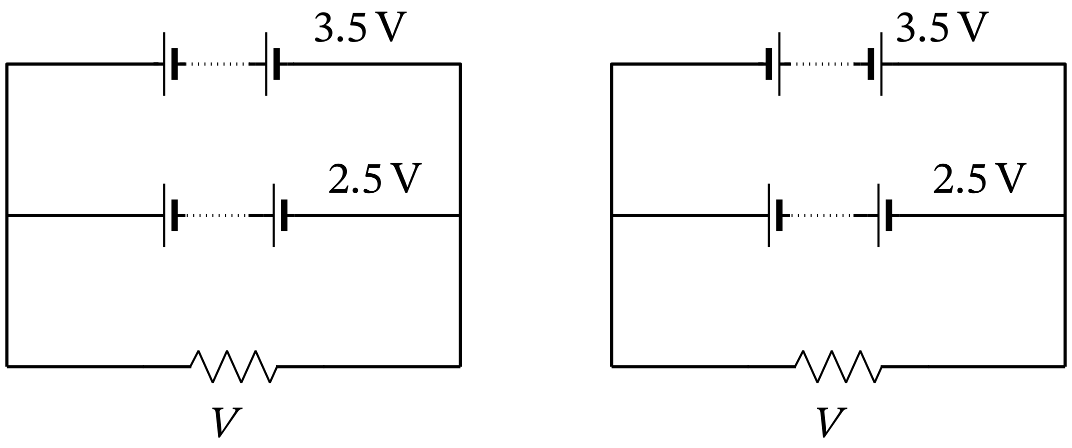


If the magnitude of V_1 is not equal to that of V_2, then the loop appears to not follow Kirchhoff's second law; this is not possible, as energy must always be conserved! In that case, there is an internal resistance in each battery that must be considered that causes the drop in potential required to conserve energy in the circuit.

So, when batteries in parallel do not have identical voltage, internal resistance must be known to be able to analyze them.

Example 4: Kirchhoff's Second Law Applied to Batteries in Parallel

The resistor in the circuit shown is powered by two batteries in parallel that are combined in two different configurations. The batteries have terminal voltages of 3.5 V and 2.5 V respectively. In the first configuration, the positive terminals of the battery are directly connected to each other and the negative terminals are connected to each other. In the second configuration, the positive terminals of each battery are directly connected to the negative terminal of the other battery. Which of the following is a correct statement about how the decrease in potential across the resistor compares in the two configurations?



Example 4 (Continued)

A. The decrease in potential will be the same in both configurations.
B. The decrease in potential in both configurations will depend on the internal resistances of the batteries.
C. The decrease in potential will be greater in the second configuration.
D. The decrease in potential will be greater in the first configuration.

Answer

First, we will consider the top circuit. Applying Kirchhoff's second law to the loop containing the top battery and the resistor gives

$$3.5\,\mathrm{V} - V = 0.$$

Applying Kirchhoff's second law to the loop containing the bottom battery and the resistor gives

$$2.5\,\mathrm{V} - V = 0.$$

Example 4 (Continued)

The two expressions give different answers! Energy must be conserved in the circuit, so there must be some internal resistance in each battery that must be known to be able to analyze the circuit correctly.

We can repeat the same process on the bottom circuit. Applying Kirchhoff's second law to the loop containing the top battery and the resistor gives

$$3.5\,\mathrm{V} - V = 0.$$

Repeating this for the loop containing the bottom battery and the resistor gives

$$-2.5\,\mathrm{V} - V = 0.$$

Again, the two expressions produce different answers. This means that the internal resistance of the batteries must be known to correctly apply Kirchhoff's laws to the circuit.

The only way to know the exact effect of changing the configuration of the batteries is if the internal resistances of the batteries are known. Therefore, the answer is B.

Key Points

- Energy is conserved in a circuit. This means that, for a charged particle moving around the circuit, the work done on the particle is equal to the change in electrical potential energy of the particle.
- Kirchhoff's first law states that the sum of the currents into a junction/node in a circuit, $I_{1,\text{in}} + I_{2,\text{in}} + \cdots$, must be the same as the sum of the currents out of the junction/node, $I_{1,\text{out}} + I_{2,\text{out}} + \cdots$:
$$I_{1,\text{in}} + I_{2,\text{in}} + \cdots = I_{1,\text{out}} + I_{2,\text{out}} + \cdots .$$
- Kirchhoff's second law states that the sum of the potential difference across each component in a loop, $V_1, V_2, \dots, V_N$, is equal to zero:
$$V_1 + V_2 + \cdots + V_N = 0.$$
- For cells connected in parallel which have different terminal voltages or which have different terminal polarities, the currents produced in circuit branches by the cells can only be determined if the internal resistances of the cells are known.