Explainer: Resistance and Resistivity of Conductors

In this explainer, we will learn how to relate the dimensions of and the motion of free electrons through an object to its resistance.

The electrical resistance of an object is given by the following formula.

Formula: Electrical Resistance

For an object that has a potential difference, V, across it and a current, I, through it, the resistance, R, of the object is given by

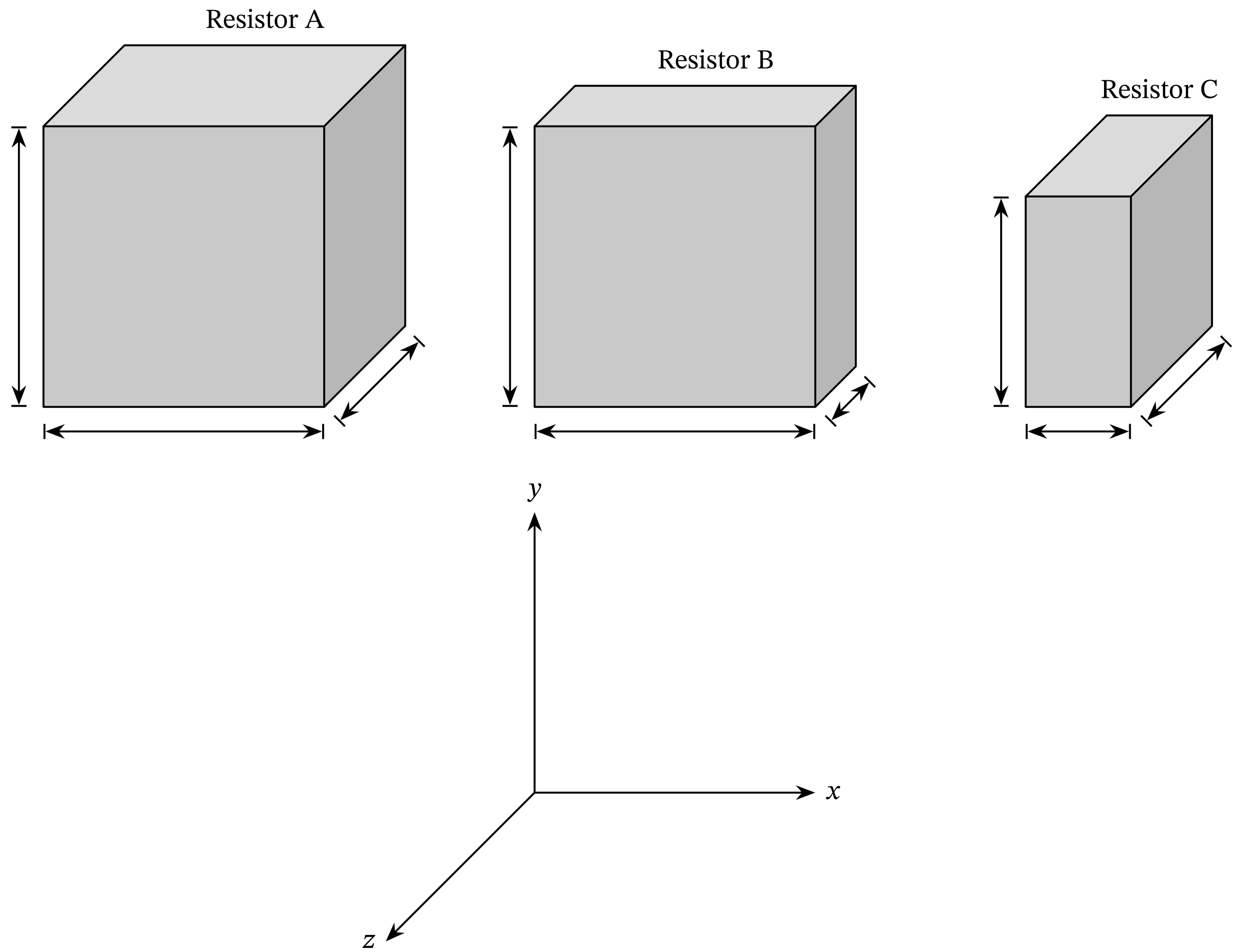
$$R = \frac{V}{I}.$$

Resistance is a property of an object. The resistance of an object depends on two factors:

- the dimensions of the object,
- a property of the substance that the object consists of, called the resistivity of the substance.

Let us first consider how the dimensions of an object affect the resistance of the object.

The following figure shows three resistors. The area of the side of the resistors in the xy-plane is the cross-sectional area of each resistor.



Resistor A and resistor B have the same cross-sectional area. This is greater than the cross-sectional area of resistor C.

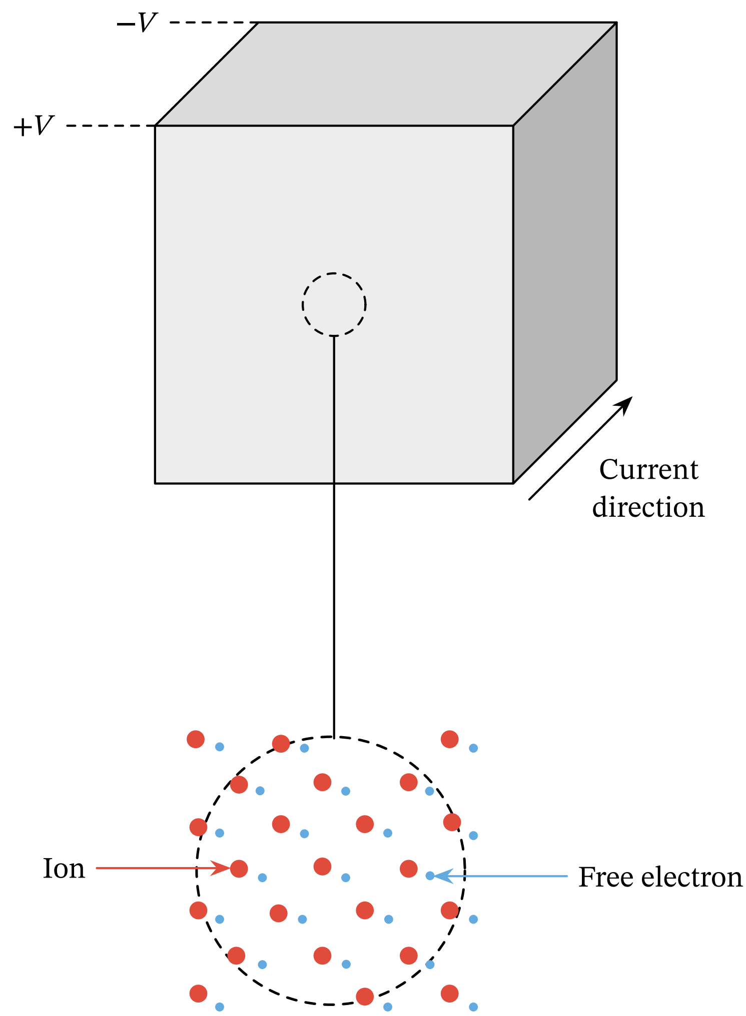
Resistor A and resistor C have the same length in the *z*-direction. This is greater than the length of resistor B.

The length and the cross-sectional area of a resistor affects how free electrons pass through the resistor. To understand this, it is useful to consider a model of the internal structure of a resistor.

A resistor that is made of an electrically conductive metal consists of a lattice of atoms that have one or more electrons in their outer orbits that are very weakly bound to the nucleus of the atom and can be pushed from one such atom to another by a small electrical force.

The resistor can be modeled as consisting of positively charged ions and free electrons that pass between the ions. The free electrons can be modeled as moving in a way similar to the motion of particles of a gas.

The following figure represents the surface of the cross section of a resistor that has a potential difference applied to it perpendicularly to the cross-sectional area.



The magnitude of the current due to the applied potential difference is given by the following formula.

Formula: Electric Current

For an object in which a charge, Q, passes a point on the object in time t, the current, I, in the object is given by

$$I = \frac{Q}{t}.$$

The charges that move through the resistor are the free electrons.

The figure representing the structure of the resistor shows us that the greater the cross-sectional area of the resistor, the greater the number of free electrons that can occupy that area.

We can therefore modify the formula for electric current to the form

$$I \propto A \times \frac{1}{t},$$

where A is the cross-sectional area of the resistor.

The time taken, t, for a free electron to move the length of the resistor is given by

$$t = \frac{l}{v},$$

where v is the average speed of free electrons and l is the length of the resistor.

We can therefore modify the formula for electric current to the form

$$I \propto A \times \frac{v}{l}$$
$$I \propto \frac{Av}{l}.$$

We have seen that

$$R = \frac{V}{I}.$$

For a constant potential difference, this is can be expressed as

$$R \propto \frac{1}{I}.$$

We can substitute the expression for I into this expression for R. This gives us

$$R \propto \frac{l}{Av}.$$

This can be expressed in a form that isolates the quantities that correspond to dimensions of a resistor as follows:

$$R \propto \frac{1}{v} \times \frac{l}{A}.$$

We see then that the effect of the dimensions of the resistor affect the resistance of a resistor in the following ways:

- the resistance of a resistor is directly proportional to the length of the resistor,
- the resistance of a resistor is inversely proportional to the cross-sectional area of the resistor.

As mentioned earlier, the factor affecting the resistance of a resistor that is due to the substance of the resistor rather than its dimensions is called the resistivity of the substance. The symbol ρ is used for resistivity.

As we have seen that

$$R \propto \frac{1}{v} \times \frac{l}{A},$$

we can see that

$$\rho \propto \frac{1}{v}.$$

The greater the resistivity of a substance, the slower free electrons move through the substance.

The resistance of a resistor can be expressed exactly using resistivity as follows:

$$R = \frac{\rho l}{A}.$$

The resistivity of a substance depends on two quantities:

- the rate at which free electrons can move through a substance,
- the density of free electrons in a substance.

We can see from this that the rate at which free electrons can move through a substance and the density of free electrons in a substance depend on each other.

The formula relating resistance and resistivity can be rearranged to make resistivity the subject, as follows:

$$R \times \frac{A}{l} = \frac{\rho l}{A} \times \frac{A}{l} = \rho.$$

From this, we can define resistivity.

Definition: Resistivity

For an object with resistance R, a cross-sectional area A, and a length l, the resistivity, ρ, is given by

$$\rho = \frac{RA}{l}.$$

The SI unit of the quantity given by

$$\frac{A}{l}$$

is given by

$$\frac{\mathrm{m}^2}{\mathrm{m}} = \mathrm{m};$$

hence, the SI unit of resistivity is given by

$$\Omega\cdot\mathrm{m}.$$

This is written in words as *ohm-metre*.

The resistivity of a substance varies with temperature. For objects of most substances, the resistivity increases as the temperature of the object increases. There is an increase in the rate of collisions between the ions and free electrons as temperature increases, which reduces the net motion of free electrons through a conductor. The mechanism for this is described at the end of the explainer.

Let us look at an example in which the resistivity of a substance is determined.

Example 1: Determining the Resistivity of a Substance

A wire made of an unknown substance has a resistance of 125 mΩ. The wire has a length of 1.8 m and a cross-sectional area of 2.35×10^{-5} m^2. What is the resistivity of the substance from which the wire is made? Give your answer in scientific notation to one decimal place.

Answer

The resistivity, ρ, of the substance is given by the formula

$$\rho = \frac{RA}{l},$$

where R is the resistance of the wire, A is the cross-sectional area of the wire, and l is the length of the wire.

Substituting the values given in the question, we find that

$$\rho = \frac{125 \times 10^{-3}\ \Omega \times 2.35 \times 10^{-5}\ \mathrm{m}^2}{1.8\ \mathrm{m}}.$$

This can be written as

$$\rho = \frac{0.125\ \Omega \times 2.35 \times 10^{-5}\ \mathrm{m}^2}{1.8\ \mathrm{m}}.$$

This can then be written as

$$\rho = \frac{0.125\ \Omega \times 2.35 \times 10^{-5}\ \mathrm{m}^2}{1.8\ \mathrm{m}} = \frac{2.9375 \times 10^{-6}\ \Omega\cdot\mathrm{m}^2}{1.8\ \mathrm{m}}.$$

To one decimal place, this is

$$1.6 \times 10^{-6}\ \Omega\cdot\text{m}.$$

Let us look at an example in which the dimensions of a resistor made of a substance of known resistivity is determined.

Example 2: Determining Dimensions of a Resistor of Known Resistivity

A copper wire with a resistance of 12.8 mΩ has a cross-sectional area of 1.15×10^{-5} m^2. Find the length of the wire. Use 1.7×10^{-8} Ω·m for the resistivity of copper. Give your answer to one decimal place.

Answer

The resistivity, ρ, of the substance is given by the formula

$$\rho = \frac{RA}{l},$$

where R is the resistance of the wire, A is the cross-sectional area of the wire, and l is the length of the wire.

This formula can be rearranged to make l the subject as follows:

$$\rho \times l = \frac{RA}{l} \times l = RA$$
$$\rho l = RA$$
$$\frac{\rho l}{\rho} = \frac{RA}{\rho} = l$$
$$l = \frac{RA}{\rho}.$$

Substituting the values given in the question, we find that

$$l = \frac{12.8 \times 10^{-3}\ \Omega \times 1.15 \times 10^{-5}\ \text{m}^2}{1.7 \times 10^{-8}\ \Omega\cdot\text{m}}$$
$$l = \frac{1.28 \times 10^{-2}\ \Omega \times 1.15 \times 10^{-5}\ \text{m}^2}{1.7 \times 10^{-8}\ \Omega\cdot\text{m}}$$
$$l = \frac{1.472 \times 10^{-7}\ \Omega\cdot\text{m}^2}{1.7 \times 10^{-8}\ \Omega\cdot\text{m}}$$
$$l = \frac{1.472 \times 10^{-7}}{1.7 \times 10^{-8}}\ \text{m}.$$

To one decimal place, this is 8.7 m.

Let us look at another example in which the dimensions of a resistor made of a substance of a known resistivity is determined.

Example 3: Determining Dimensions of a Resistor of Known Resistivity

A copper wire with a resistance of 22 mΩ has a length of 6.2 m. Find the cross-sectional area. Use 1.7×10^{-8} Ω⋅m for the resistivity of copper. Give your answer in scientific notation to one decimal place.

Answer

The resistivity, ρ, of the substance is given by the formula

$$\rho = \frac{RA}{l},$$

where R is the resistance of the wire, A is the cross-sectional area of the wire, and l is the length of the wire.

This formula can be rearranged to make A the subject as follows:

$$\begin{aligned}
\rho\times l &= \frac{RA}{l}\times l = RA\\
\rho l &= RA\\
\frac{\rho l}{R} &= \frac{RA}{R} = A\\
A &= \frac{\rho l}{R}.
\end{aligned}$$

Substituting the values given in the question, we find that

$$\begin{aligned}
A &= \frac{1.7\times10^{-8}\ \Omega\cdot\mathrm{m}\times6.2\ \mathrm{m}}{22\times10^{-3}\ \Omega}\\
A &= \frac{1.7\times10^{-8}\ \Omega\cdot\mathrm{m}\times6.2\ \mathrm{m}}{2.2\times10^{-2}\ \Omega}\\
A &= \frac{1.054\times10^{-7}\ \Omega\cdot\mathrm{m}^2}{2.2\times10^{-2}\ \Omega}.
\end{aligned}$$

To one decimal place, this is

$$4.8\times10^{-6}\ \mathrm{m}^2.$$

We have seen that the charge that moves past a point on a resistor in a time is given by

$$Q = It.$$

The charge that moves past a point on a resistor in a time is given by

$$Q = e \times N,$$

where e is the charge of an electron and N is the number of electrons that move past the point.

The value of N depends on the density of free electrons in a substance, n, and the volume of a resistor made of that substance. For a uniform resistor, the resistor volume is the product of its length and its cross-sectional area. We see then that

$$N = n \times A \times l.$$

The charge that moves past a point on a resistor in a time can now be written as

$$Q = neAl.$$

Dividing both sides of the equation by the time for which charge moves, we obtain

$$\frac{Q}{t} = \frac{neAl}{t}.$$

We can see that

$$\frac{l}{t} = v,$$

where v is the average speed at which electrons travel across the resistor. The term used for v is the *drift velocity of free electrons*.

We know that

$$\frac{Q}{t} = I;$$

hence, we obtain a formula relating the current in a resistor to the average speed at which electrons travel across the resistor.

Formula: Current in Terms of the Drift Velocity of Free Electrons

For a resistor made of a substance with a free electron density n that has a cross-sectional area A and carries a current I,

$$I = neAv,$$

where e is the charge of an electron and v is the drift velocity of free electrons in the resistor.

Let us now look at an example in which a drift velocity is determined.

Example 4: Determining the Drift Velocity of Free Electrons

A current of 1.4 A in a copper wire is carried by free electrons. The cross-sectional area of the wire is 2.5×10^{-6} m^2. Find the average speed at which free electrons pass through the wire. Use a value of 1.6×10^{-19} C for electron charge and a value of 8.46×10^{28} m^{-3} for the density of free electrons in copper. Give your answer in scientific notation to one decimal place.

Answer

The current in the wire is related to the average speed of free electrons by

$$I = neAv,$$

where n is the free electron density of copper, e is the charge of an electron, A is the cross-sectional area of the wire, and v is the drift velocity of free electrons in the wire.

The drift velocity can be made the subject of the equation as follows:

$$\frac{I}{neA} = \frac{neAv}{neA} = v.$$

Substituting the values given in the question, we find that

$$v = \frac{1.4\ \text{A}}{8.46 \times 10^{28}\ \text{m}^{-3} \times 1.6 \times 10^{-19}\ \text{C} \times 2.5 \times 10^{-6}\ \text{m}^2}$$

$$v = \frac{1.4\ \frac{\text{C}}{\text{s}}}{33\,840\ \frac{\text{C}}{\text{m}}}$$

$$v = \frac{1.4}{33\,840}\ \frac{\text{m}}{\text{s}}.$$

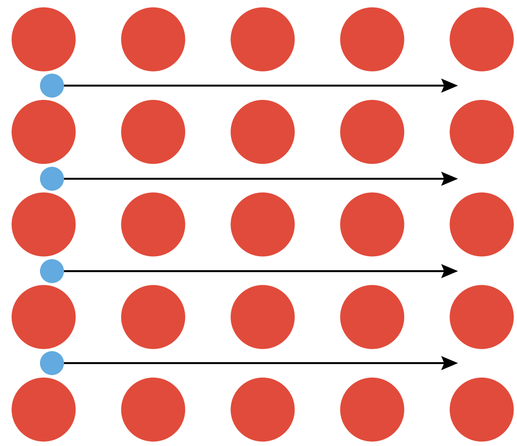
To one decimal place, this is

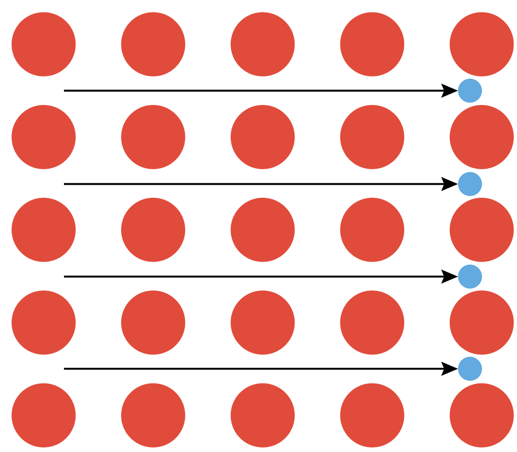
$$4.1 \times 10^{-5}\ \text{m/s}.$$

The drift velocity of free electrons is surprisingly small.

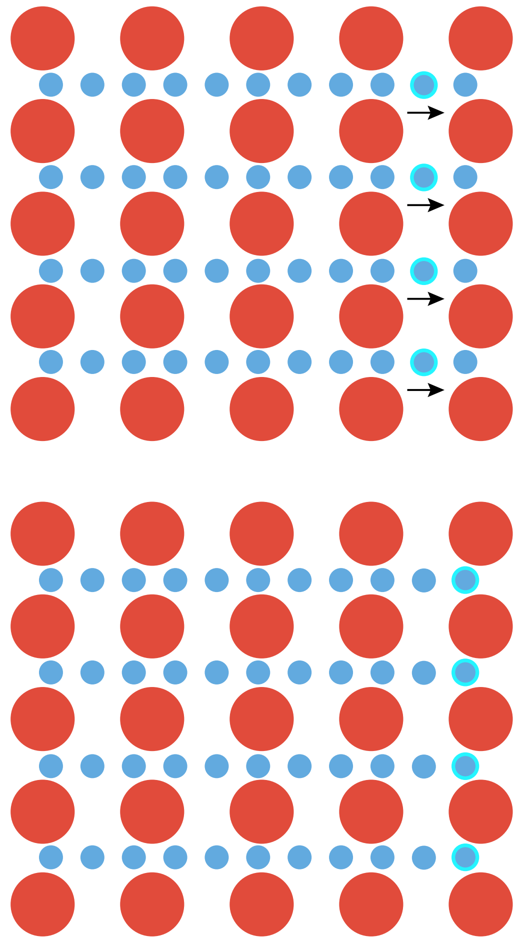
When an electric circuit is closed, the current in the circuit is present throughout the circuit almost immediately. A delay is not detectable to human senses. This might make someone suppose that individual free electrons must move across the length of the circuit in negligible time.

The following figure represents the incorrect understanding of the motion of free electrons across a circuit.





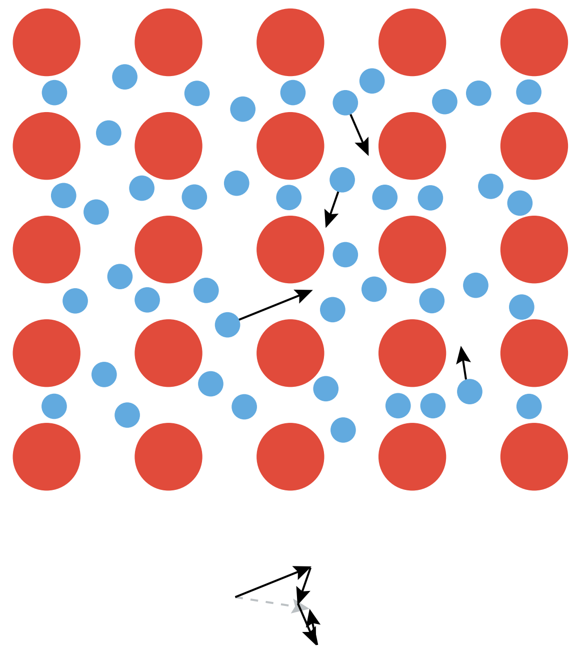
It is important to notice that, in this incorrect model, free electrons are only present at the starting and ending points of a circuit. In reality, free electrons are present throughout the circuit. This is shown in the following figure.



The figure shows the highlighted free electrons moving much slower than in the incorrect model, but there are many more free electrons moving.

The free electrons in a conductor do not actually move in the uniform way suggested by the preceding diagrams. The motion shown in these diagrams is the net motion of electrons rather than the motions of individual electrons.

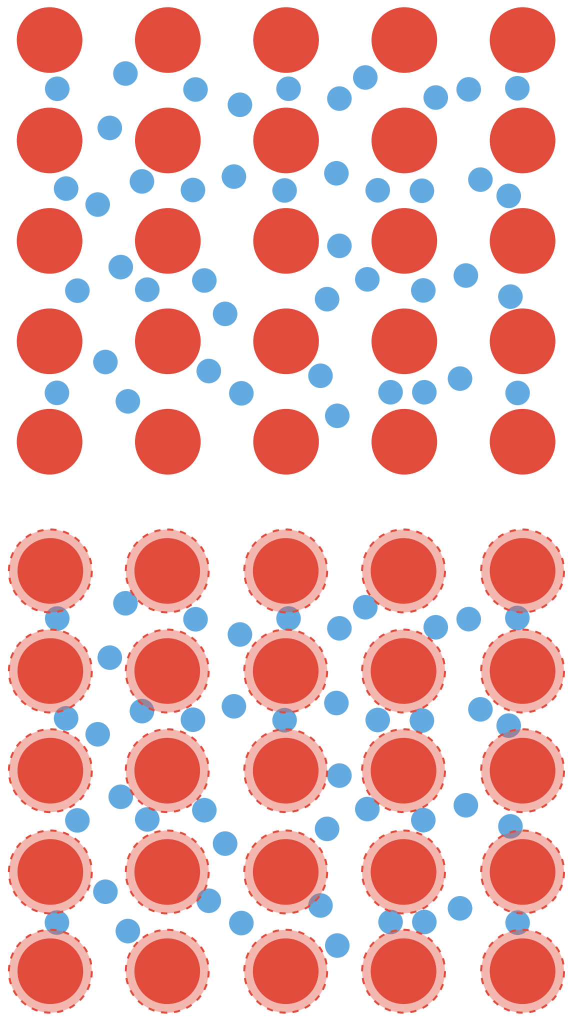
The following figure better represents the motions of individual electrons. Only the motion of four individual electrons, selected randomly, is shown.



We see that while only some of the electrons have a velocity with a positive component in the direction of the current, the net velocity of these electrons (shown by the gray dashed vector) has such a component.

Understanding that the motions of individual electrons differ considerably from the direction of the net motion of electrons helps explain why resistivity tends to increase with temperature.

The following figure represents the same conductor at two different temperatures.



Increased temperature

At a higher temperature, an ion in the conductor will tend to undergo greater changes in displacement around its average position than at a lesser temperature. The range of possible positions of the ions therefore increases, as shown in the figure.

This means that collisions between ions and electrons become more likely. The more collisions that occur between ions and electrons, the more the current in the conductor is reduced and hence the more the resistivity of the conductor increases.

Let us now summarize what has been learned in this explainer.

Key Points

- The resistance of an object depends on the dimensions of the object and a property of the substance that the object consists of called the resistivity of the substance.

- For an object with resistance R, a cross-sectional area A, and length l, the resistivity, ρ, is given by

$$\rho = \frac{RA}{l}.$$

- Resistivity has the unit ohm-metre ($\Omega \cdot \mathrm{m}$).
- The greater the resistivity of a substance, the more energy required to produce a current in an object made from the substance.
- The resistivity of a substance is related to the density of free electrons in the substance.
- The resistivity of a substance is related to the average velocity at which free electrons move through the substance.
- The resistivities of most substances increase as temperature increases.
- For a resistor made of a substance with a free electron density, n, that has a cross-sectional area, A, and carries a current, I,

$$I = neAv,$$

  where e is the charge of an electron and v is the drift velocity of free electrons in the resistor.
- The time taken for a free electron to travel the length of a circuit is generally much greater than the time taken to establish a current through the circuit.