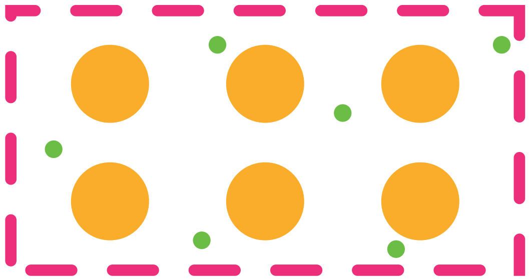
Resistance and Resistivity of Conductors

$$R = \frac{\rho L}{A}$$



Lesson Objectives

You will be able to

- use the formula $R = \dfrac{\rho l}{A}$ to relate the resistivity, resistance, and dimensions of a resistor,
- diagrammatically represent, in terms of the motion of free electrons, the effects of varying the length, cross-sectional area, and resistivity of a resistor on its resistance,
- diagrammatically represent, in terms of the motion of free electrons, the effects of varying the temperature of resistors,
- use the formula $I = nAv_de$ to relate the current in a resistor to the drift speed of free electrons in the resistor.

Recap: Electric Current

For an object in which a charge, Q, passes a point on the object in time t, the current, I, in the object is given by

$$I = \frac{Q}{t}.$$

The current in an object depends on the potential difference across the object and the resistance of the object.

Recap: Resistance

For an object that has a potential difference, V, across it and a current, I, through it, the resistance, R, of the object is given by

$$R = \frac{V}{I}.$$

Resistance is a property of an object. The resistance of an object depends on two factors:

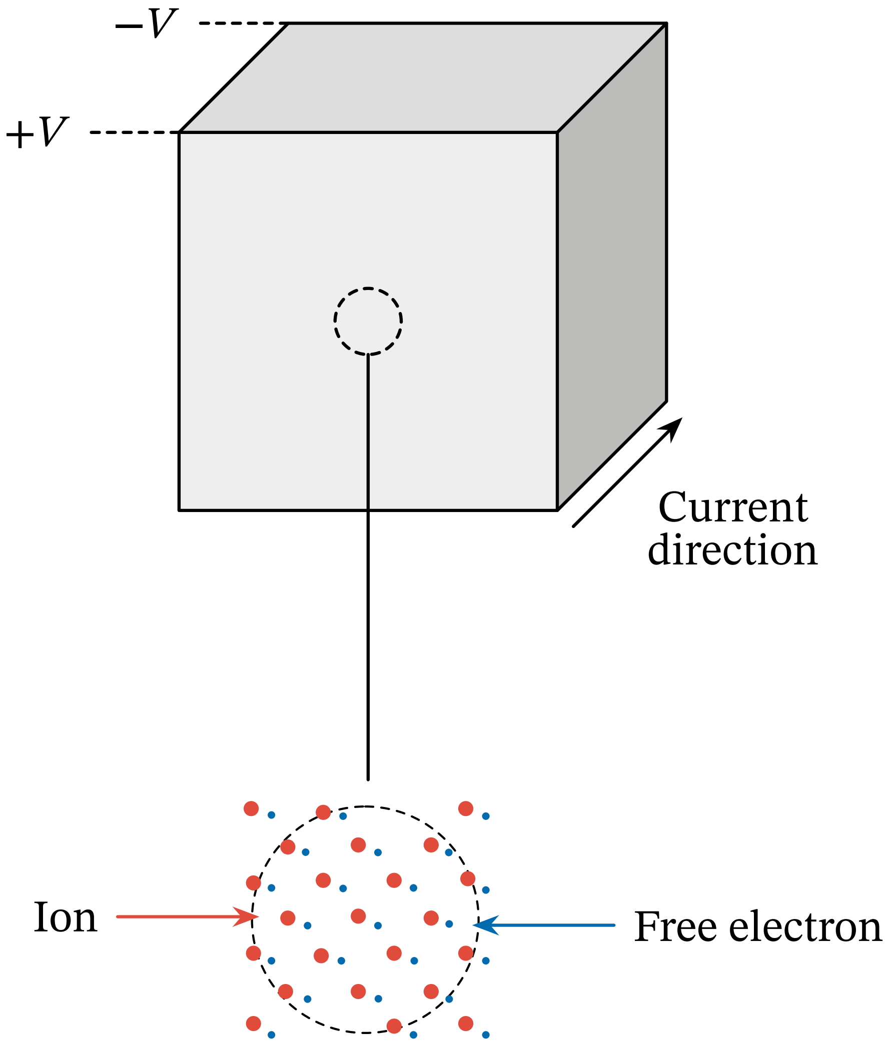
- the dimensions of the object,
- a property of the substance that the object consists of, called the resistivity of the substance, ρ.

The Motion of Free Electrons through a Resistor

A resistor that is made of an electrically conductive metal consists of a lattice of atoms that have one or more electrons in their outer orbits that are very weakly bound to the nucleus of the atom and can be pushed from one such atom to another by a small electrical force.

The resistor can be modeled as consisting of positively charged ions and free electrons that pass between the ions. The free electrons can be modeled as moving in a way similar to the motion of particles of a gas.

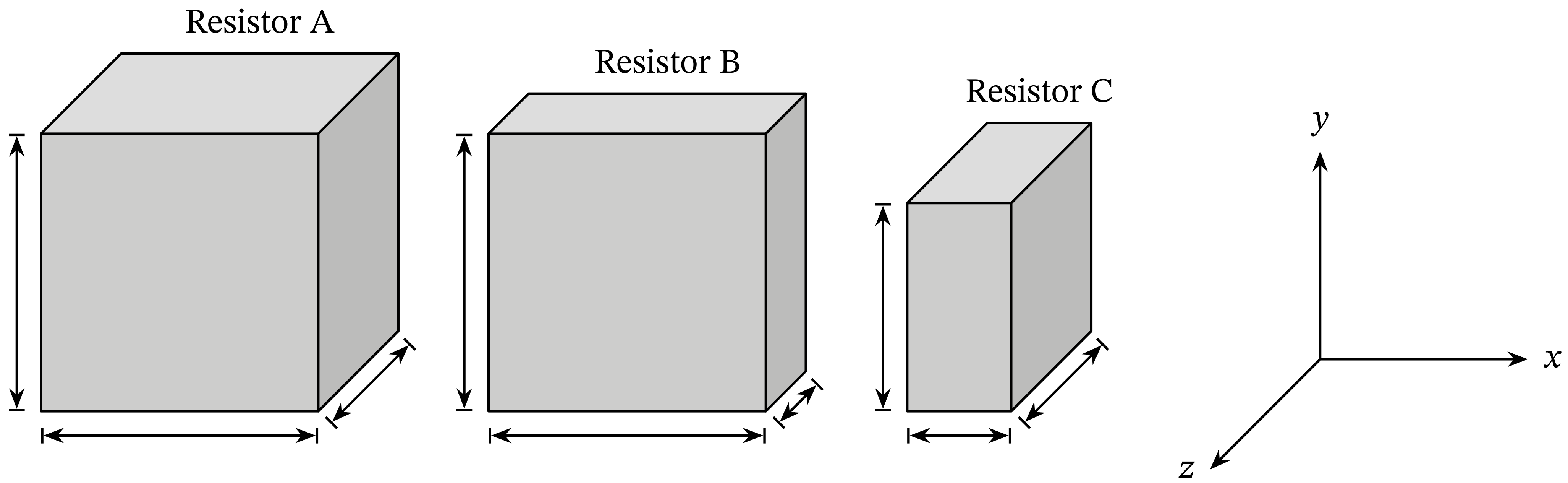
The greater the cross-sectional area of the resistor, the greater the number of free electrons that can occupy that area.



The Effect of Object Dimensions on Its Resistance

The length and the cross-sectional area of a resistor affect how free electrons pass through the resistor.

The area of the side of the three resistors shown in the *xy*-plane is the cross-sectional area of each resistor.



Resistor A and resistor B have the same cross-sectional area. This is greater than the cross-sectional area of resistor C.

Resistor A and resistor C have the same length in the *z*-direction. This is greater than the length of resistor B.

The Effect of Object Dimensions on Its Resistance (Continued)

Taking into account the relationship between the cross-sectional area of a resistor and the number of free electrons that contribute to the current in the resistor, we can express the formula

$$I = \frac{Q}{t}$$

as

$$I \propto A \times \frac{1}{t},$$

where A is the cross-sectional area of the resistor.

The time taken, t, for a free electron to move the length of the resistor is given by

$$t = \frac{l}{v},$$

where v is the average speed of free electrons and l is the length of the resistor.

This expression for t can be substituted into the equation

$$I \propto A \times \frac{1}{t}$$

The Effect of Object Dimensions on Its Resistance (Continued)

to give

$$I \propto A \times \frac{v}{l}$$

$$I \propto \frac{Av}{l}.$$

We have seen that

$$R = \frac{V}{I}.$$

For a constant potential difference, this can be expressed as

$$R \propto \frac{1}{I}.$$

We can substitute into this the expression

$$I \propto \frac{Av}{l}$$

The Effect of Object Dimensions on Its Resistance (Continued)

to give

$$R \propto \frac{l}{Av}.$$

The expression

$$R \propto \frac{l}{Av}$$

can be written in a form that isolates the quantities that correspond to the dimensions of a resistor as follows:

$$R \propto \frac{1}{v} \times \frac{l}{A}.$$

We see then that the dimensions of the resistor affect its resistance in the following ways:

- the resistance of a resistor is directly proportional to the length of the resistor;
- the resistance of a resistor is inversely proportional to the cross-sectional area of the resistor.

The Effect of the Resistivity of a Substance on the Resistance of an Object

In the equation

$$R \propto \frac{1}{v} \times \frac{l}{A},$$

only l and A correspond to the dimensions of a resistor.

The $\frac{1}{v}$ term of the equation relates to the resistivity, ρ, of the substance that a resistor is made of.

We see then that

$$\rho \propto \frac{1}{v}.$$

The greater the resistivity of a substance is, the slower the free electrons move through the substance.

The resistance of a resistor can be expressed exactly using resistivity as follows:

$$R = \frac{\rho l}{A}.$$

The Effect of the Resistivity of a Substance on the Resistance of an Object (Continued)

The resistivity of a substance depends on two quantities:

- the rate at which free electrons can move through a substance,
- the density of free electrons in a substance.

We can see from this that the rate at which free electrons can move through a substance and the density of free electrons in a substance depend on each other.

Definition of Resistivity

The formula

$$R = \frac{\rho l}{A}$$

can be rearranged to make resistivity the subject as follows:

$$R \times \frac{A}{l} = \frac{\rho l}{A} \times \frac{A}{l}$$

$$\rho = \frac{RA}{l}.$$

The SI unit of the quantity given by

$$\frac{A}{l}$$

is given by

$$\frac{\mathrm{m}^2}{\mathrm{m}} = \mathrm{m};$$

hence, the SI unit of resistivity is given by

$$\Omega \cdot \mathrm{m}.$$

This is written in words as *ohm-metre*.

Example 1: Determining the Resistivity of a Substance

A wire made of an unknown substance has a resistance of 125 mΩ. The wire has a length of 1.8 m and a cross-sectional area of $2.35 \times 10^{-5}\ \mathrm{m}^2$. What is the resistivity of the substance from which the wire is made? Give your answer in scientific notation to one decimal place.

Answer

The resistivity, ρ, of the substance is given by the formula

$$\rho = \frac{RA}{l},$$

where R is the resistance of the wire, A is the cross-sectional area of the wire, and l is the length of the wire.

$$\rho = \frac{125 \times 10^{-3}\ \Omega \times 2.35 \times 10^{-5}\ \mathrm{m}^2}{1.8\ \mathrm{m}}$$

First, we substitute the values given in the question.

$$\rho = \frac{2.9375 \times 10^{-6}\ \Omega \cdot \mathrm{m}^2}{1.8\ \mathrm{m}}$$

We then simplify the numerator.

To one decimal place, this is $1.6 \times 10^{-6}\ \Omega \cdot \mathrm{m}$.

Example 2: Determining Dimensions of a Resistor of Known Resistivity

A copper wire with a resistance of 12.8 mΩ has a cross-sectional area of $1.15 \times 10^{-5}\ \mathrm{m}^2$. Find the length of the wire. Use $1.7 \times 10^{-8}\ \Omega \cdot \mathrm{m}$ for the resistivity of copper. Give your answer to one decimal place.

Answer

The resistivity, ρ, of the substance is given by the formula

$$\rho = \frac{RA}{l},$$

where R is the resistance of the wire, A is the cross-sectional area of the wire, and l is the length of the wire.

This formula can be rearranged to make l the subject as follows:

$$\rho \times l = \frac{RA}{l} \times l$$

$$\rho \times l = RA$$

$$\frac{RA}{\rho} = l.$$

Example 2 (Continued)

Substituting the known values into

$$\frac{RA}{\rho} = l,$$

we have

$$l = \frac{12.8 \times 10^{-3}\,\Omega \times 1.15 \times 10^{-5}\,\mathrm{m}^2}{1.7 \times 10^{-8}\,\Omega \cdot \mathrm{m}}$$

$$l = \frac{0.128\,\Omega \times 1.15 \times 10^{-5}\,\mathrm{m}^2}{1.7 \times 10^{-8}\,\Omega \cdot \mathrm{m}}$$

$$l = \frac{1.472 \times 10^{-7}\,\Omega \cdot \mathrm{m}^2}{1.7 \times 10^{-8}\,\Omega \cdot \mathrm{m}}$$

$$l = \frac{1.472 \times 10^{-7}\,\mathrm{m}}{1.7 \times 10^{-8}}.$$

To one decimal place, this is 8.7 m.

Example 3: Determining Dimensions of a Resistor of Known Resistivity

A copper wire with a resistance of $22\ \mathrm{m\Omega}$ has a length of $6.2\ \mathrm{m}$. Find the cross-sectional area of the wire. Use $1.7 \times 10^{-8}\ \Omega \cdot \mathrm{m}$ for the resistivity of copper. Give your answer to one decimal place.

Answer

The resistivity, ρ, of the substance is given by the formula

$$\rho = \frac{RA}{l},$$

where R is the resistance of the wire, A is the cross-sectional area of the wire, and l is the length of the wire.

This formula can be rearranged to make A the subject as follows:

$$\begin{aligned} \rho \times l &= \frac{RA}{l} \times l \\ \rho \times l &= RA \\ \rho \times \frac{l}{R} &= A. \end{aligned}$$

Example 3 (Continued)

Substituting the known values into

$$A = \rho \times \frac{l}{R},$$

we have

$$A = \frac{1.7 \times 10^{-8}\,\Omega \cdot \mathrm{m} \times 6.2\,\mathrm{m}}{22 \times 10^{-3}\,\Omega}$$

$$A = \frac{1.7 \times 10^{-8}\,\Omega \cdot \mathrm{m} \times 6.2\,\mathrm{m}}{2.2 \times 10^{-2}\,\Omega}$$

$$A = \frac{1.054 \times 10^{-7}\,\Omega \cdot \mathrm{m}^2 \times 6.2\,\mathrm{m}}{2.2 \times 10^{-2}\,\Omega}.$$

To one decimal place, this is $4.8 \times 10^{-6}\,\mathrm{m}^2$.

Electron Drift Velocity

As well as considering the relationship between the resistance of the resistor, the resistivity of the resistor material, and the dimensions of the resistor, we can consider the relationship between these quantities and the speed at which charge-carrying free electrons move through a resistor.

As

$$Q = It,$$

the charge that moves past a point on a resistor in a time is given by

$$Q = e \times N,$$

where e is the charge of an electron and N is the number of electrons that move past the point.

The value of N depends on the density of free electrons in a substance, n, and the volume of a resistor made of that substance. For a uniform resistor, the resistor volume is the product of its length and its cross-sectional area. We see then that

$$N = n \times A \times l.$$

Electron Drift Velocity (Continued)

The charge that moves past a point on a resistor in a time can now be written as

$$Q = neAl.$$

Dividing both sides of the equation by the time for which the charge moves, we obtain

$$\frac{Q}{t} = \frac{neAl}{t}.$$

We can see then that

$$\frac{l}{t} = v,$$

where v is the average speed at which electrons travel across the resistor.

The term used for v is the *drift velocity of free electrons*.

Electron Drift Velocity (Continued)

For a resistor made of a substance with a free electron density n that has a cross-sectional area A and carries a current I,

$$I = neAv,$$

where e is the charge of an electron and v is the drift velocity of free electrons in the resistor.

Let us now look at an example in which drift velocity is determined.

Example 4: Determining the Drift Velocity of Free Electrons

A current of 1.4 A in a copper wire is carried by free electrons. The cross-sectional area of the wire is $2.5 \times 10^{-6}\ \mathrm{m}^2$. Find the average speed at which free electrons pass through the wire. Use a value of 1.6×10^{-19} C for electron charge and a value of $8.46 \times 10^{28}\ \mathrm{m}^{-3}$ for the density of free electrons in copper. Give your answer in scientific notation to one decimal place.

Answer

The current in the wire is related to the average speed of free electrons by

$$I = neAv,$$

where e is the charge of an electron and v is the drift velocity of free electrons in the resistor.

The drift velocity can be made the subject of the equation as follows:

$$\frac{I}{neA} = \frac{neAv}{neA}$$
$$\frac{neAv}{neA} = v$$
$$\frac{I}{neA} = v.$$

Example 4 (Continued)

Substituting the known values into

$$v = \frac{I}{neA},$$

we have

$$\begin{aligned} v &= 1.4 \times 8.46 \times 10^{28}\,\mathrm{m}^{-3} \times 1.6 \times 10^{-19}\,\mathrm{C} \times 2.5 \times 10^{-6}\,\mathrm{m}^2 \\ v &= \frac{1.4\,\mathrm{C/s}}{33\,840\,\mathrm{C/m}} \\ v &= \frac{1.4\,\mathrm{m}}{33\,840\,\mathrm{s}}. \end{aligned}$$

To one decimal place, this is 4.1×10^{-5} m/s.

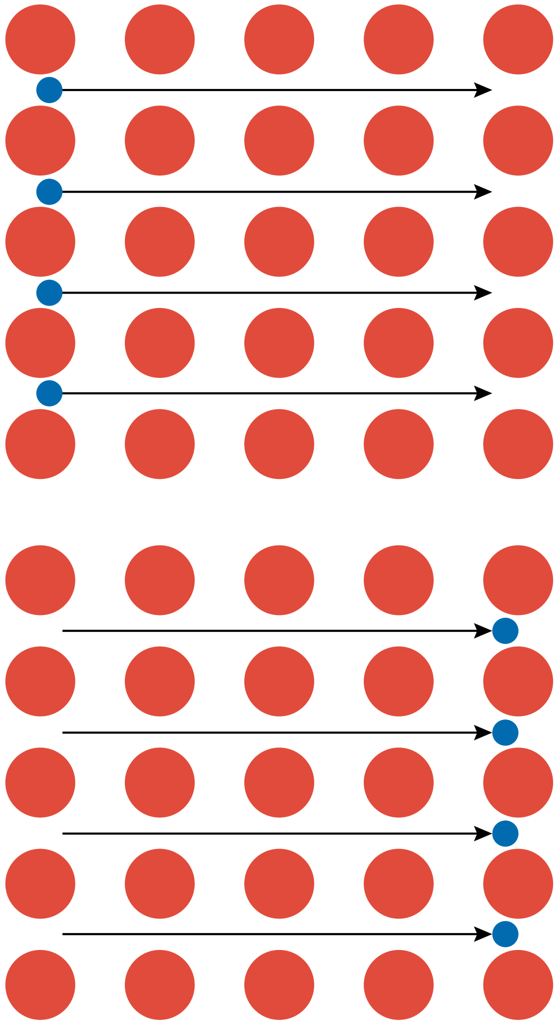
Electron Drift Velocity (Continued)

The drift velocity of free electrons is surprisingly small.

When an electric circuit is closed, the current in the circuit is present throughout the circuit almost immediately. A delay is not detectable to human senses. This might make someone suppose that individual free electrons must move across the length of the circuit in negligible time.

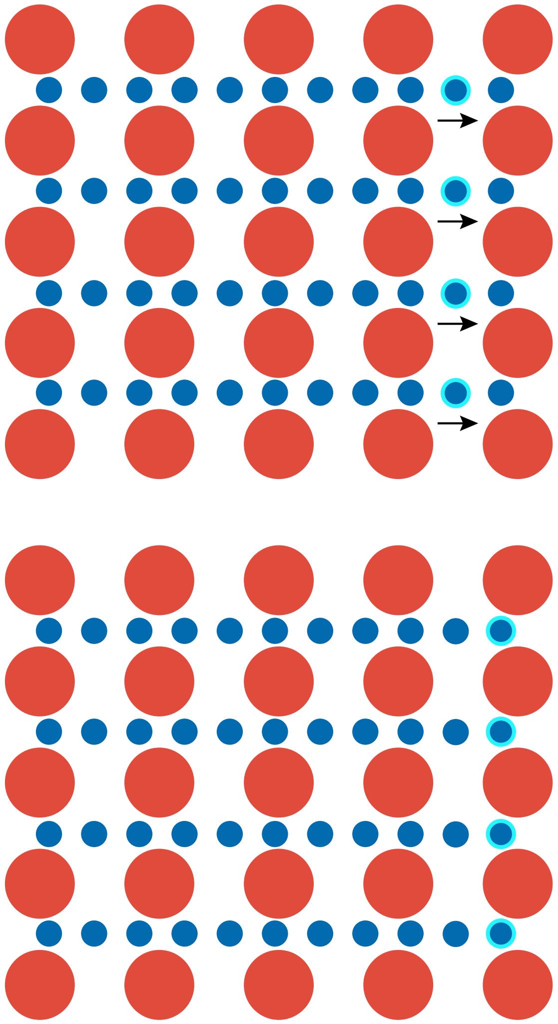
In fact, individual free electrons must only *pass a point*, not cross the length of a circuit. This is because free electrons are present throughout the circuit.

Electron Drift Velocity (Continued)



Distance moved by free electrons in a unit time is large.

Incorrect representation of free electron motion in a resistor

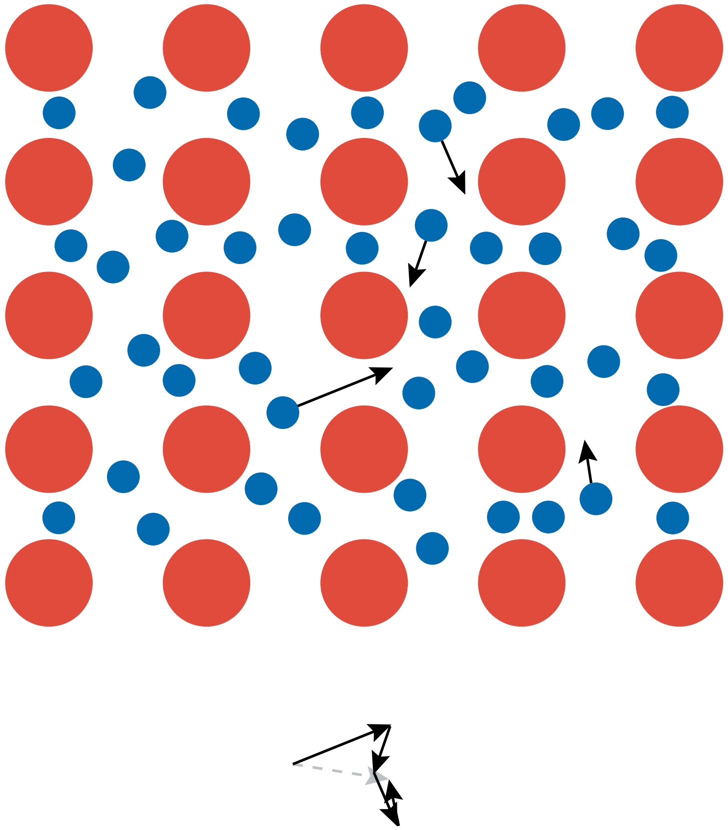


Distance moved by free electrons in a unit time is small.

Correct representation of free electron motion in a resistor

Electron Drift Velocity (Continued)

The free electrons in a conductor do not actually move in a uniform way. The following figure better represents the motions of individual electrons. Only the motion of four individual electrons, selected randomly, is shown.

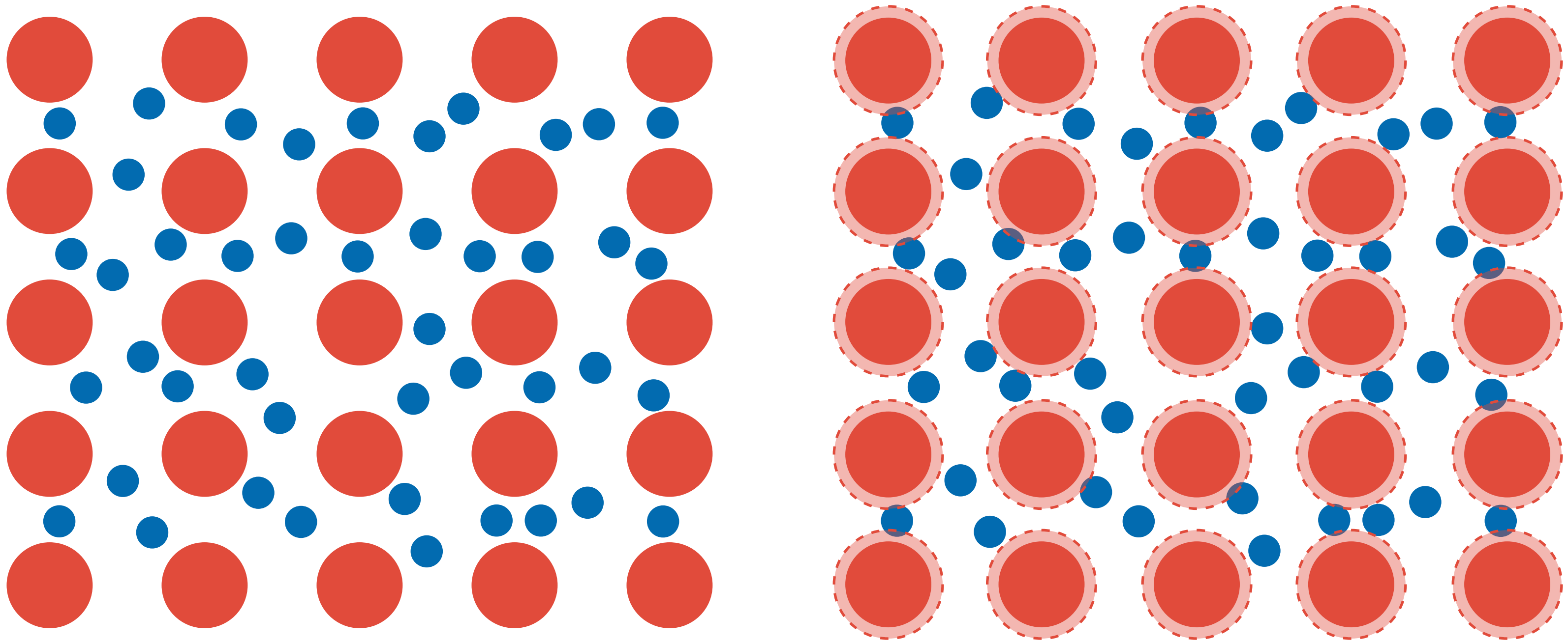


We see that while only some of the electrons have a velocity with a positive component in the direction of the current, the net velocity of these electrons (shown by the gray dashed vector) has such a component.

Effect of Temperature on Resistivity

The resistivity of a substance varies with temperature. For most substances, the resistivity increases as the temperature increases.

There is an increase in the rate of collisions between the ions and free electrons as temperature increases, which reduces the net motion of free electrons through a conductor.



Increased temperature

Effect of Temperature on Resistivity (Continued)

At a higher temperature, an ion in the conductor will tend to undergo greater changes in displacement around its average position than at a lesser temperature. The range of possible positions of the ions therefore increases, as shown in the figure.

This means that collisions between ions and electrons become more likely.

The more the collisions occur between ions and electrons, the more the current in the conductor is reduced and hence the more the resistivity of the conductor increases.

Key Points

- The resistance of an object depends on the dimensions of the object and a property of the substance that the object consists of called the resistivity of the substance.
- For an object with resistance R, a cross-sectional area A, and length l, the resistivity, ρ, is given by
$$\rho = \frac{RA}{l}.$$
- Resistivity has the unit ohm-metre ($\Omega \cdot \mathrm{m}$).
- The greater the resistivity of a substance, the more energy required to produce a current in an object made from the substance.
- The resistivity of a substance is related to the density of free electrons in the substance.
- The resistivity of a substance is related to the average velocity at which free electrons move through the substance.
- The resistivities of most substances increase as temperature increases.

Key Points (Continued)

- For a resistor made of a substance with a free electron density, n, that has a cross-sectional area, A, and carries a current, I,

  $$I = neAv,$$

  where e is the charge of an electron and v is the drift velocity of free electrons in the resistor.
- The time taken for a free electron to travel the length of a circuit is generally much greater than the time taken to establish a current through the circuit.